LESSON 8.3

Distance Between Two Points

TEKS **Expressions, equations, and relationships—8.7.D** Determine the distance between two points on a coordinate plane using the Pythagorean Theorem.

ESSENTIAL QUESTION

How can you use the Pythagorean Theorem to find the distance between two points on a coordinate plane?

Pythagorean Theorem in the Coordinate Plane

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EXAMPLE 1

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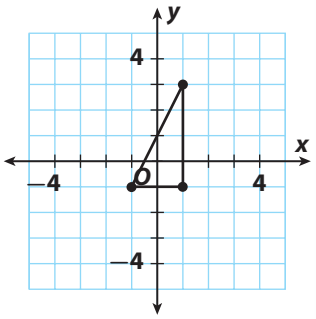
The figure shows a right triangle. Approximate the length of the hypotenuse to the nearest tenth without using a calculator.



STEP 1 Find the length of each leg.

The length of the vertical leg is 4 units.

The length of the horizontal leg is 2 units.

STEP 2 Let $a = 4$ and $b = 2$. Let c represent the length of the hypotenuse. Use the Pythagorean Theorem to find c.

$a^2 + b^2 = c^2$

$4^2 + 2^2 = c^2$ Substitute into the formula.

$20 = c^2$ Add.

$\sqrt{20} = c$ Take the square root of both sides.

STEP 3 Approximate $\sqrt{20}$ by finding perfect squares close to 20.

$\sqrt{20}$ is between $\sqrt{16}$ and $\sqrt{25}$, or $\sqrt{16} < \sqrt{20} < \sqrt{25}$.

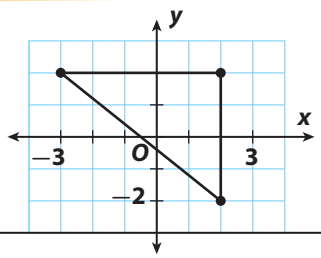
Simplifying gives $4 < \sqrt{20} < 5$.

Since 20 is about halfway between 16 and 25, $\sqrt{20}$ is about halfway between 4 and 5. So, $\sqrt{20} \approx 4.5$.

The hypotenuse is about 4.5 units long.

YOUR TURN

1. Approximate the length of the hypotenuse to the nearest tenth without using a calculator.



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EXPLORE ACTIVITY

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Finding the Distance Between Any Two Points

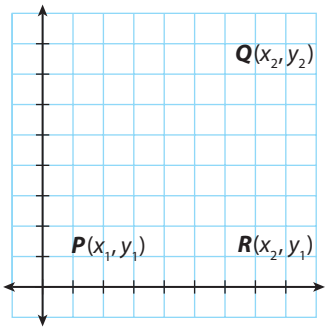
The Pythagorean Theorem can be used to find the distance between any two points (x_1, y_1) and (x_2, y_2) in the coordinate plane. The resulting expression is called the Distance Formula.

Distance Formula

In a coordinate plane, the distance d between two points (x_1, y_1) and (x_2, y_2) is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

Use the Pythagorean Theorem to derive the Distance Formula.



A To find the distance between points P and Q, draw segment $\overline{PQ}$ and label its length d. Then draw horizontal segment $\overline{PR}$ and vertical segment $\overline{QR}$. Label the lengths of these segments a and b. Triangle PQR is a _____ triangle, with hypotenuse _____.

B Since $\overline{PR}$ is a horizontal segment, its length, a, is the difference between its x-coordinates. Therefore, $a = x_2 -$ _____.

C Since $\overline{QR}$ is a vertical segment, its length, b, is the difference between its y-coordinates. Therefore, $b = y_2 -$ _____.

D Use the Pythagorean Theorem to find d, the length of segment $\overline{PQ}$. Substitute the expressions from **B** and **C** for a and b.

$d^2 = a^2 + b^2$

$d = \sqrt{a^2 + b^2}$

$d = \sqrt{\left(\square - \square\right)^2 + \left(\square - \square\right)^2}$

Math Talk
Mathematical Processes

What do $x_2 - x_1$ and $y_2 - y_1$ represent in terms of the Pythagorean Theorem?

Reflect

2. Why are the coordinates of point R the ordered pair (x_2, y_1)?

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Finding the Distance Between Two Points

The Pythagorean Theorem can be used to find the distance between two points in a real-world situation. You can do this by using a coordinate grid that overlays a diagram of the real-world situation.

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EXAMPLE 2

Real World

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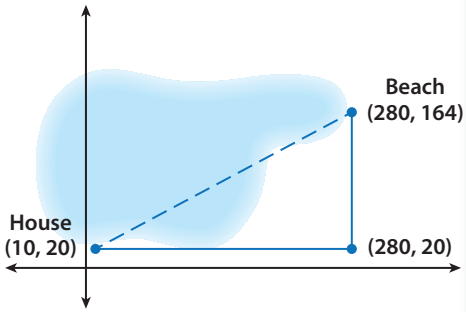
Francesca wants to find the distance between her house on one side of a lake and the beach on the other side. She marks off a third point forming a right triangle, as shown. The distances in the diagram are measured in meters.



Use the Pythagorean Theorem to find the straight-line distance from Francesca's house to the beach.

STEP 1 Find the length of the horizontal leg.

The length of the horizontal leg is the absolute value of the difference between the x-coordinates of the points (280, 20) and (10, 20).

$$|280 - 10| = 270$$

The length of the horizontal leg is 270 meters.

STEP 2 Find the length of the vertical leg.

The length of the vertical leg is the absolute value of the difference between the y-coordinates of the points (280, 164) and (280, 20).

$$|164 - 20| = 144$$

The length of the vertical leg is 144 meters.

STEP 3 Let $a = 270$ and $b = 144$. Let c represent the length of the hypotenuse. Use the Pythagorean Theorem to find c.

$a^2 + b^2 = c^2$

$270^2 + 144^2 = c^2$ Substitute into the formula.

$72{,}900 + 20{,}736 = c^2$ Simplify.

$93{,}636 = c^2$ Add.

$\sqrt{93{,}636} = c$ Take the square root of both sides.

$306 = c$ Simplify.

The distance from Francesca's house to the beach is 306 meters.

Math Talk
Mathematical Processes

Why is it necessary to take the absolute value of the coordinates when finding the length of a segment?

Reflect

3. Show how you could use the Distance Formula to find the distance from Francesca's house to the beach.

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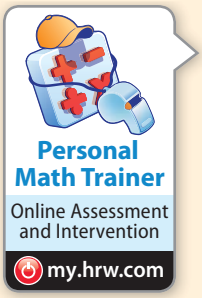

YOUR TURN

4. Camp Sunshine is also on the lake. Use the Pythagorean Theorem to find the distance between Francesca's house and Camp Sunshine to the nearest tenth of a meter.

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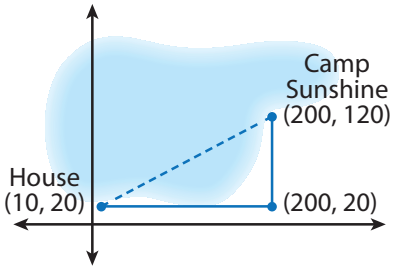
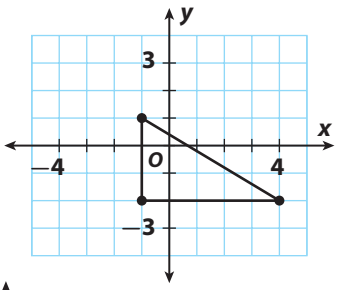


Guided Practice

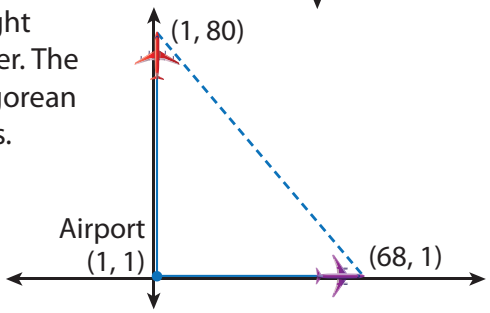
1. Approximate the length of the hypotenuse of the right triangle to the nearest tenth without using a calculator. (Example 1) _____



2. Find the distance between the points $(3, 7)$ and $(15, 12)$ on the coordinate plane. (Explore Activity) _____

3. A plane leaves an airport and flies due north. Two minutes later, a second plane leaves the same airport flying due east. The flight plan shows the coordinates of the two planes 10 minutes later. The distances in the graph are measured in miles. Use the Pythagorean Theorem to find the distance shown between the two planes.

(Example 2) _____



ESSENTIAL QUESTION CHECK-IN

4. Describe two ways to find the distance between two points on a coordinate plane.

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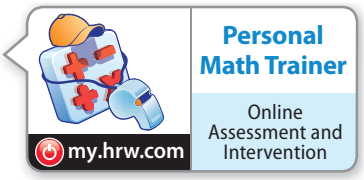
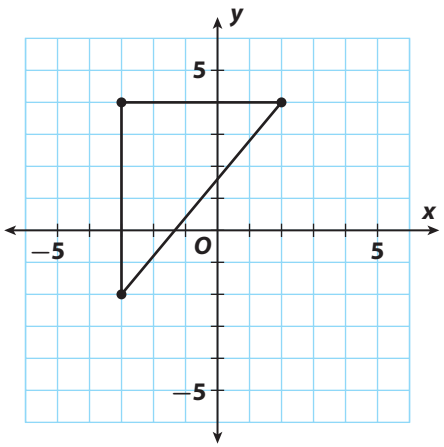
Name _____ Class _____ Date _____

8.3 Independent Practice

TEKS 8.7.D



5. A metal worker traced a triangular piece of sheet metal on a coordinate plane, as shown. The units represent inches. What is the length of the longest side of the metal triangle? Approximate the length to the nearest tenth of an inch without using a calculator.



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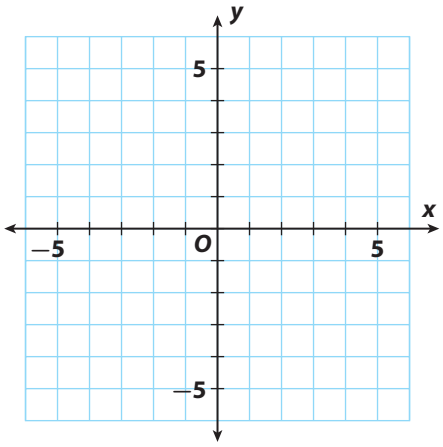
6. When a coordinate grid is superimposed on a map of Harrisburg, the high school is located at (17, 21) and the town park is located at (28, 13). If each unit represents 1 mile, how many miles apart are the high school and the town park? Round your answer to the nearest tenth.

_____

7. The coordinates of the vertices of a rectangle are given by $R(-3, -4)$, $E(-3, 4)$, $C(4, 4)$, and $T(4, -4)$. Plot these points on the coordinate plane at the right and connect them to draw the rectangle. Then connect points E and T to form diagonal $\overline{ET}$.



a. Use the Pythagorean Theorem to find the exact length of $\overline{ET}$.

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b. How can you use the Distance Formula to find the length of $\overline{ET}$? Show that the Distance Formula gives the same answer.

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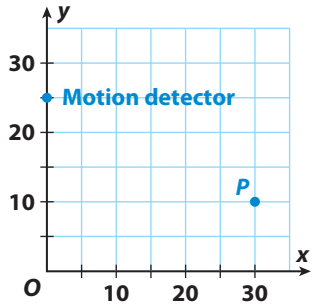
8. **Multistep** The locations of three ships are represented on a coordinate grid by the following points: $P(-2, 5)$, $Q(-7, -5)$, and $R(2, -3)$. Which ships are farthest apart?

_____

9. **Make a Conjecture** Find as many points as you can that are 5 units from the origin. Make a conjecture about the shape formed if all the points 5 units from the origin were connected.

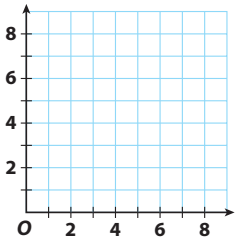
10. **Justify Reasoning** The graph shows the location of a motion detector that has a maximum range of 34 feet. A peacock at point *P* displays its tail feathers. Will the motion detector sense this motion? Explain.



Work Area

H.O.T. FOCUS ON HIGHER ORDER THINKING

11. **Persevere in Problem Solving** One leg of an isosceles right triangle has endpoints (1, 1) and (6, 1). The other leg passes through the point (6, 2). Draw the triangle on the coordinate plane below. Then show how you can use the Distance Formula to find the length of the hypotenuse. Round your answer to the nearest tenth.



12. **Represent Real-World Problems** The figure shows a representation of a football field. The units represent yards. A sports analyst marks the locations from where the football was thrown (point *A*) and where it was caught (point *B*). Explain how you can use the Pythagorean Theorem to find the distance the ball was thrown. Then find the distance.