LESSON

12.1 Properties of Translations

TEKS Two-dimensional shapes—8.10.A
Generalize the properties of orientation and congruence of . . . translations . . . of two-dimensional shapes on a coordinate plane.

ESSENTIAL QUESTION

How do you describe the properties of orientation and congruence of translations?

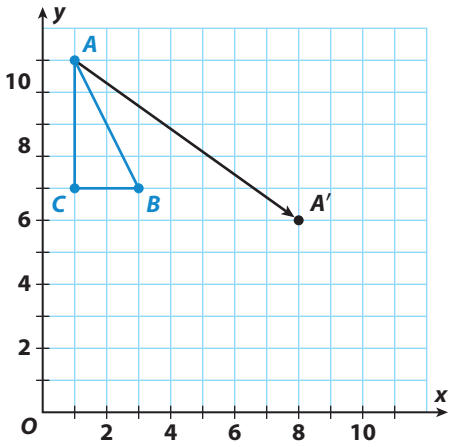
EXPLORE ACTIVITY 1 TEKS 8.10.A

Exploring Translations

You learned that a function is a rule that assigns exactly one output to each input. A **transformation** is a function that describes a change in the position, size, or shape of a figure. The input of a transformation is the **preimage**, and the output of a transformation is the **image**.

A **translation** is a transformation that slides a figure along a straight line. The image has the same size and shape as the preimage.

The triangle shown on the grid is the preimage (input). The arrow shows the motion of a translation and how point *A* is translated to point *A′*.



A Trace triangle *ABC* onto a piece of paper. Cut out your traced triangle.

B Slide your triangle along the arrow to model the translation that maps point *A* to point *A′*.

C The image of the translation is the triangle produced by the translation. Sketch the image of the translation.

D The vertices of the image are labeled using prime notation. For example, the image of *A* is *A′*. Label the images of points *B* and *C*.

E Describe the motion modeled by the translation.

Move _____ units right and _____ units down.

F Check that the motion you described in part **E** is the same motion that maps point *A* onto *A′*, point *B* onto *B′*, and point *C* onto *C′*.

Reflect

1. How is the orientation of the triangle affected by the translation?

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EXPLORE ACTIVITY 2

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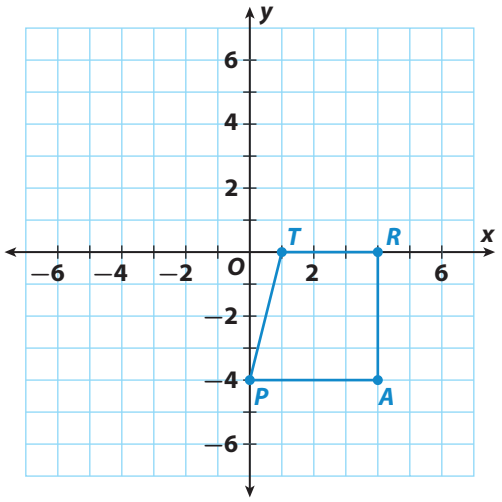
Properties of Translations

Use trapezoid *TRAP* to investigate the properties of translations.



A Trace the trapezoid onto a piece of paper. Cut out your traced trapezoid.

B Place your trapezoid on top of the trapezoid in the figure. Then translate your trapezoid 5 units to the left and 3 units up. Sketch the image of the translation by tracing your trapezoid in this new location. Label the vertices of the image T', R', A', and P'.

C Use a ruler to measure the sides of trapezoid *TRAP* in centimeters.

$TR =$ _____ $RA =$ _____ $AP =$ _____ $TP =$ _____

D Use a ruler to measure the sides of trapezoid $T'R'A'P'$ in centimeters.

$T'R' =$ _____ $R'A' =$ _____ $A'P' =$ _____ $T'P' =$ _____

E What do you notice about the lengths of corresponding sides of the two figures?

_____

F Use a protractor to measure the angles of trapezoid *TRAP*.

$m\angle T =$ _____ $m\angle R =$ _____ $m\angle A =$ _____ $m\angle P =$ _____

G Use a protractor to measure the angles of trapezoid $T'R'A'P'$.

$m\angle T' =$ _____ $m\angle R' =$ _____ $m\angle A' =$ _____ $m\angle P' =$ _____

H What do you notice about the measures of corresponding angles of the two figures?

_____

I Which sides of trapezoid *TRAP* are parallel? How do you know?

_____

Which sides of trapezoid $T'R'A'P'$ are parallel? _____

What do you notice? _____

_____

Reflect

2. **Make a Conjecture** Use your results from parts E, H, and I to make a conjecture about translations.

_____

_____

3. What can you say about translations and congruence?

_____

_____

Graphing Translations

To translate a figure in the coordinate plane, translate each of its vertices. Then connect the vertices to form the image.

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EXAMPLE 1

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The figure shows triangle *XYZ*. Graph the image of the triangle after a translation of 4 units to the right and 1 unit up.

STEP 1 Translate point *X*.

Count right 4 units and up 1 unit and plot point *X′*.

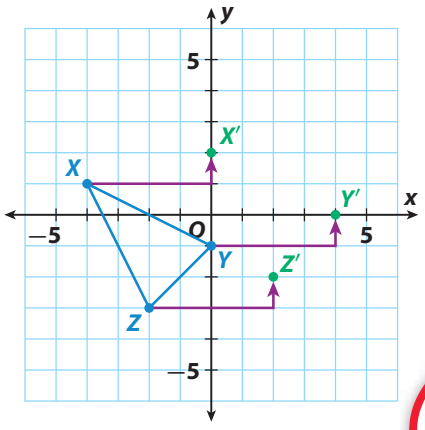
STEP 2 Translate point *Y*.

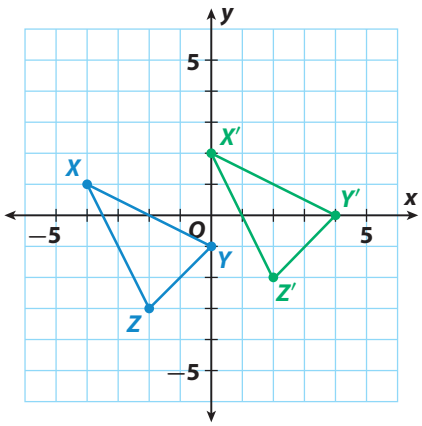
Count right 4 units and up 1 unit and plot point *Y′*.

STEP 3 Translate point *Z*.

Count right 4 units and up 1 unit and plot point *Z′*.

STEP 4 Connect *X′*, *Y′*, and *Z′* to form triangle *X′Y′Z′*.





Each vertex is moved 4 units right and 1 unit up.

Math Talk
Mathematical Processes
Is the image congruent to the preimage? How do you know?

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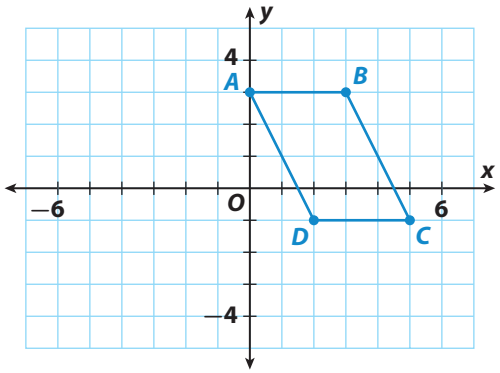
YOUR TURN

4. The figure shows parallelogram *ABCD*. Graph the image of the parallelogram after a translation of 5 units to the left and 2 units down.



Guided Practice

1. **Vocabulary** A _____ is a change in the position, size, or shape of a figure.

2. **Vocabulary** When you perform a transformation of a figure on the coordinate plane, the input of the transformation is called the _____, and the output of the transformation is called the _____.

3. Joni translates a right triangle 2 units down and 4 units to the right. How does the orientation of the image of the triangle compare with the orientation of the preimage? (Explore Activity 1)

_____

4. Rashid drew rectangle *PQRS* on a coordinate plane. He then translated the rectangle 3 units up and 3 units to the left and labeled the image *P′Q′R′S′*. How do rectangle *PQRS* and rectangle *P′Q′R′S′* compare? (Explore Activity 2)

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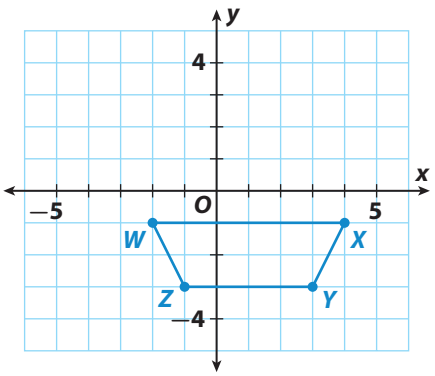
5. The figure shows trapezoid *WXYZ*. Graph the image of the trapezoid after a translation of 4 units up and 2 units to the left. (Example 1)



ESSENTIAL QUESTION CHECK-IN

6. What are the properties of translations?

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12.1 Independent Practice

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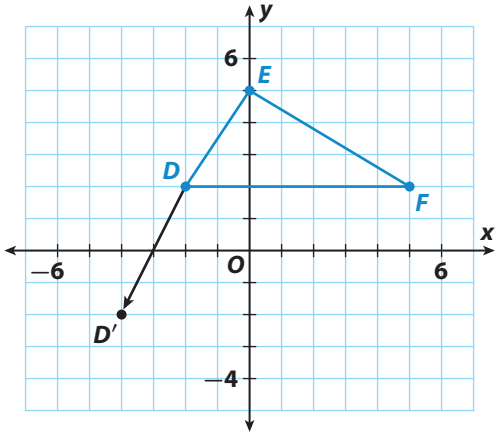
7. The figure shows triangle *DEF*.



a. Graph the image of the triangle after the translation that maps point *D* to point *D′*.

b. How would you describe the translation?

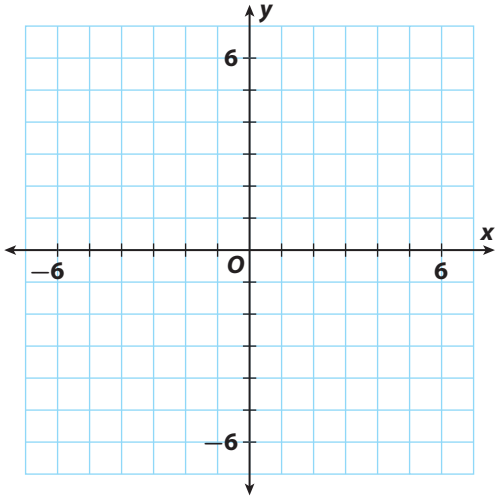
_____

_____

c. How does the image of triangle *DEF* compare with the preimage?

_____

8. **a.** Graph quadrilateral *KLMN* with vertices *K*(–3, 2), *L*(2, 2), *M*(0, –3), and *N*(–4, 0) on the coordinate grid.



b. On the same coordinate grid, graph the image of quadrilateral *KLMN* after a translation of 3 units to the right and 4 units up.

c. Which side of the image is congruent to side $\overline{LM}$?

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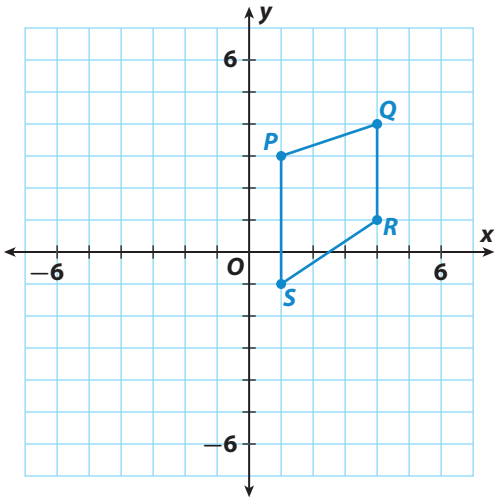
Name three other pairs of congruent sides.

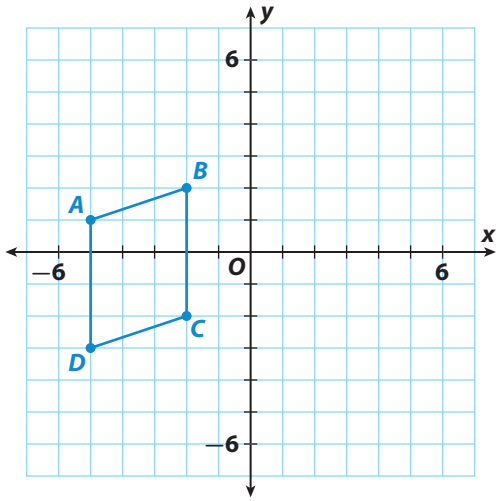
_____

Draw the image of the figure after each translation.

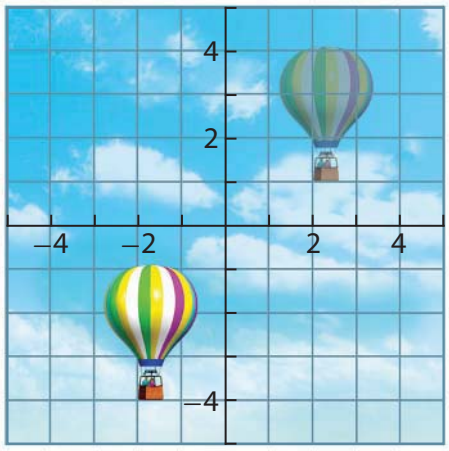
9. 4 units left and 2 units down



10. 5 units right and 3 units up



11. The figure shows the ascent of a hot air balloon. How would you describe the translation?



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12. **Critical Thinking** Is it possible that the orientation of a figure could change after it is translated? Explain.

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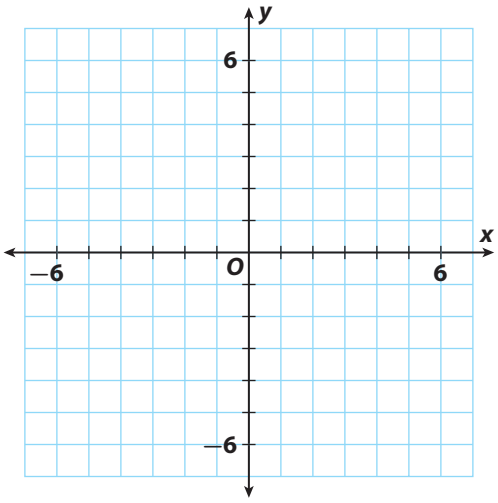
H.O.T. FOCUS ON HIGHER ORDER THINKING

13. **a.** **Multistep** Graph triangle XYZ with vertices $X(-2, -5)$, $Y(2, -2)$, and $Z(4, -4)$ on the coordinate grid.

b. On the same coordinate grid, graph and label triangle $X'Y'Z'$, the image of triangle XYZ after a translation of 3 units to the left and 6 units up.

c. Now graph and label triangle $X''Y''Z''$, the image of triangle $X'Y'Z'$ after a translation of 1 unit to the left and 2 units down.

d. **Analyze Relationships** How would you describe the translation that maps triangle XYZ onto triangle $X''Y''Z''$?



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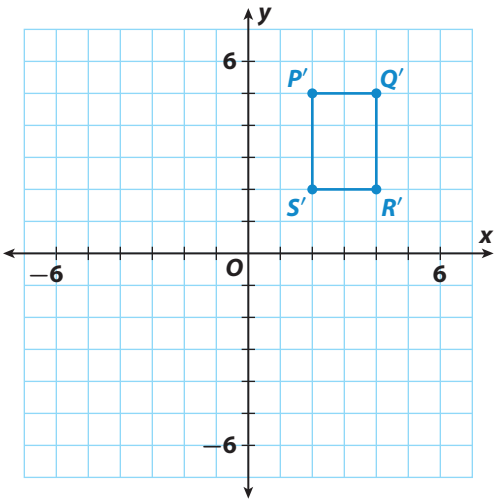
14. **Critical Thinking** The figure shows rectangle $P'Q'R'S'$, the image of rectangle $PQRS$ after a translation of 5 units to the right and 7 units up. Graph and label the preimage $PQRS$.

15. **Communicate Mathematical Ideas** Explain why the image of a figure after a translation is congruent to its preimage.



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